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# Input-output signal selection for damping of power system oscillations using wind power plants

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## Abstract

During the last years wind power has emerged as one of the most important sources in the power generation share. Due to stringent Grid Code requirements, wind power should provide ancillary services such as fault-ride through and power oscillation damping to resemble conventional generation. A proper selection of input-output signal pairs to damp electromechanical oscillations using wind power plants is important to ensure such contribution. In this paper, different analysis techniques considering both controllability and observability measures and input-output interactions are compared. From this comparison, recommendations are drawn to select the best signal pairs to damp power system oscillations considering different approaches, such as single-input single-output and multivariable control.

*Keywords:* Controllability, damping, frequency domain analysis, observability, power system oscillation, state space representation, wind power plant.

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## 1. Introduction

As a consequence of increased wind power penetration levels, transmission system operators (TSOs) are concerned with system stability. Wind power plants (WPPs) are required by TSOs to meet Grid Code requirements, and sometimes to behave as conventional power plants being capable of providing support to the power system when it is required to maintain stability [1–3]. Power system stability is divided in three main groups depending on the response of the system to a fault: frequency, rotor angle and voltage stability [4].

Rotor angle stability is defined as the capability of synchronous generators to keep or restore the equilibrium between their mechanical and electromagnetic torques. This stability issue is usually exhibited by synchronous generators as low frequency oscillations (LFOs). The main effects of this type of oscillations are to limit the power transfer capacity of the system and to cause large grid failures. This problem used to be solved by the installation of power system stabilizers (PSSs) at synchronous generators which increase the damping of the system. However, due to recent technological advances on power system devices, damping of electromechanical oscillations has been proposed in the literature to be provided by HVDC links, energy storage systems, flexible AC transmission systems and wind power generation [5–8]. It is worth to remark that wind power is located where wind blows stronger and

is more profitable. Thus, it is difficult to locate a WPP where power oscillation damping can be better achieved [9]. The physical location of the WPP plays an important role when defining the possible input and output signals for oscillation mitigation if they are measured locally.

Different methods to select the best feedback signal to damp power oscillations have been discussed in [8, 10–14], but the case for WPPs has not been yet well covered. Recent research focuses on the best input-output signal pairs employing controllability and observability analyses such as residues and geometric measures [11, 15]. Other works study the interaction between different controllers for a multiple-input multiple-output (MIMO, multivariable) case and try to determine if a decentralized controller is possible by using the relative gain array (RGA) [15, 16]. In [17], fundamental limitations of control design by using local signals to damp remote oscillations are analyzed, where the interaction between local and remote signals has an important influence.

The aim of this paper is to compare different controllability and observability and signal interaction analyses for power system oscillation damping by using WPPs. The main advantages and drawbacks of each alternative are examined. Frequency domain tools such as the RGA and the multivariable structure function (MSF) [18–20] are employed to assess the interaction between signal pairs. For the presence of right hand plane zeros (RHPZs), the arising control design and performance limitations are clearly defined [21]. The use of some of these methods is recommended to select the best input-output pairs which ensure a good controllability and observability of the desired oscillation mode, while providing a clear insight of the po-

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output  $y_j$ . The mode  $\lambda_k$  of linear system (2) is controllable if matrix  $\mathcal{C}(\lambda_k)$  has full row rank. Similarly, the mode  $\lambda_k$  is observable if  $\mathcal{O}(\lambda_k)$  is full column rank [21].

The rank of matrices  $\mathcal{C}(\lambda_k)$  and  $\mathcal{O}(\lambda_k)$  can be evaluated by their singular values. The singular values of a matrix  $M$  are defined as  $\sigma_i = \sqrt{\lambda_k(M^T M)}$  ( $k = 1, \dots, n$ ) with  $\sigma_1 \geq \dots \geq \sigma_n \geq 0$ . The matrix rank is then given by the number of non-null singular values. In practice, due to numerical limitations, the rank is the number of singular values greater than a given tolerance. Therefore, the minimum singular values  $\sigma_n$  provide a measure of how close to singular is the matrix. In the case of matrices  $\mathcal{C}(\lambda_k)$  and  $\mathcal{O}(\lambda_k)$ ,  $\sigma_n$  indicates how far is the system of being uncontrollable or unobservable, respectively, with respect to the  $i$ -th input and  $j$ -th output.

The choice of input and output signals through the PBH test is done by selecting those with the largest of the minimum singular values  $\sigma_n$  of matrices  $\mathcal{C}(\lambda_k)$  and  $\mathcal{O}(\lambda_k)$  [10]. Although this is a reasonable input-output selection criterion to determine the most controllable and observable signals, it is not always clear which pair presents the best joint controllability and observability characteristics. From a control point of view, the joint measure is of great importance.

### 3.1.2. Residue Analysis

Given the transfer function  $g_{ij}(s)$  from the input  $u_i$  to the output  $y_j$ , it is always possible to express it as a sum of partial fractions of the form

$$g_{ij}(s) = \frac{y_j(s)}{u_i(s)} = c_j(sI - A)^{-1}b_i + d_{ij} = \sum_{k=1}^n \frac{R_k}{s - \lambda_k} + d_{ij} \quad (6)$$

where  $R_k$  is the residue associated to the mode  $\lambda_k$  [23]. The residue  $R_k$  provides an idea of how the mode  $\lambda_k$  is affected by the input  $u_i$  and how visible is from the output  $y_j$ . Therefore, the residues are clear measures of joint controllability and observability of a particular oscillation mode that is commonly used in damping oscillation analysis [4, 11, 24].

The residues can be computed directly from the state-space realization from the expression

$$R_k = c_j \phi_k \psi_k b_i \quad (7)$$

where  $\phi_k$  and  $\psi_k$  are the right and left eigenvectors of the matrix  $A$ , respectively, corresponding to the eigenvalue  $\lambda_k$ . As in general the residues are complex numbers, the best input-output signal pair is given by the maximum value of the residue magnitude. The residues depend on the scale of the input and output signals and not always provide a clear comparison among transfer functions associated to variables with different units.

### 3.1.3. Geometric Measures

The controllability and observability geometric measures, respectively, are defined as

$$m_{ci} = \cos(\theta(\psi_k, b_i)) = \frac{|b_i^T \psi_k|}{\|\psi_k\| \|b_i\|} \quad (8)$$

$$m_{oj} = \cos(\theta(\phi_k, c_j)) = \frac{|c_j \phi_k|}{\|\phi_k\| \|c_j\|} \quad (9)$$

where  $\theta(\psi_k, b_i)$  is the angle between  $b_i$  and  $\psi_k$ , and  $\theta(\phi_k, c_j)$  is the angle between  $c_j$  and  $\phi_k$ . The geometric measures provide an idea of how aligned the columns of matrix  $B$  and the rows of  $C$  are with an eigenvector of  $A$ . If  $m_{ci} = 0$ , the column of  $b_i$  is orthogonal to eigenvector  $\psi_k$  and a controller will not be effective to modify the state associated to eigenvalue  $\lambda_k$ . Similarly, if  $m_{oj} = 0$ ,  $c_j$  and  $\phi_k$  are orthogonal and mode  $\lambda_k$  will not be observable from the output  $y_j$  [25].

A joint geometric measure can be defined as

$$m_{coij} = m_{ci} m_{oj}. \quad (10)$$

A zero value of  $m_{coij}$  indicates that  $\lambda_k$  is non-controllable/non-observable from the  $i$ -th input and the  $j$ -th output. The geometric measures provide a similar information as the residues, with the advantage of being normalized and independent of the scale of the signals. These measures have been used to determine the best input-output pair in power oscillation damping applications [12, 25].

Some authors have proposed the use of Hankel singular values (HSVs) for the selection of input-output pairs [8]. However, it is not possible to connect the controllability and observability of a particular mode with the HSVs [21]. For this reason, their use is not considered in this paper.

### 3.2. Limitations caused by right hand plane zeros (RHPZs)

It is well-known that non-minimum phase zeros (zeros in the right hand plane, RHPZ) impose limitations on the achievable performance [26, 27]. To ensure closed-loop stability, the controller cannot cancel RHPZs and the frequency response of the closed-loop transfers will have fixed points that cannot be altered by the controller. For instance, the sensitivity transfer  $S(s)$  takes the value 1 at a RHPZ independently of the controller; that is, if  $\xi$  is a RHPZ, then

$$S(\xi) = (I + K(\xi)G(\xi))^{-1} = 1.$$

These constraints become serious limitations if the RHPZs are close to the mode to be damped, since the magnitude of the closed-loop transfer cannot be arbitrarily reduced. If the controller set is reduced to proportional gains, the constraints are more serious. A simple root locus analysis reveals how strong these limitations are. Since the closed-loop poles tend to the open loop zeros, if an open loop pole is close to a RHPZ, a small increment in gain may result in an unstable system and thus it is not possible to

affect the lightly damped mode. Therefore, any pair with RHPZs in the range of LFOs should be avoided.

The limitations imposed by RHPZs have been used to propose input-output selection procedures in [27, 28]. In the context of damping oscillations in power systems they have been used in [8, 17].

### 3.3. Input-output interactions

An alternative view on input-output signal pair selection can be obtained from the frequency domain. This allows to evaluate input-output interactions and thus determine if decentralized controllers will be able to achieve a reasonable closed-loop behavior.

#### 3.3.1. RGA

It allows to evaluate interactions among different inputs and outputs [21]. In the context of input-output pair selection, it helps to determine the pair with the highest interaction and to evaluate the possibility of using multivariable controllers. The RGA matrix  $H$  is defined as

$$H(s) = [\eta_{ij}(s)] = G(s) \otimes G(s)^{-T} \quad (11)$$

where  $[\eta_{ij}]$  denotes the element  $ij$  of the matrix  $H$  and the operator  $\otimes$  denotes the Hadamard or Schur product (element by element product)[21].

For example, in case of systems with two inputs and two outputs ( $2 \times 2$  system),

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (12)$$

and the RGA is given by

$$H(s) = \begin{bmatrix} \eta_{11}(s) & 1 - \eta_{11}(s) \\ 1 - \eta_{11}(s) & \eta_{11}(s) \end{bmatrix} \quad (13)$$

with

$$\eta_{11}(s) = \frac{g_{11}(s)g_{22}(s)}{g_{11}(s)g_{22}(s) - g_{12}(s)g_{21}(s)}. \quad (14)$$

The following conclusions can be drawn from the RGA (13) [18, 21].

- If  $\eta_{11}$  is close to 1, the pairs  $u_1 - y_1$  and  $u_2 - y_2$  are decoupled and can be controlled independently. In this case, a centralized multivariable controller will not achieve a better performance than the decentralized one

$$K(s) = \begin{bmatrix} k_{11}(s) & 0 \\ 0 & k_{22}(s) \end{bmatrix}, \quad (15)$$

in which  $k_{11}$  and  $k_{22}$  can be designed independently.

- If  $\eta_{11}$  is close to zero, the RGA indicates that the best pairs for decentralized control should be  $u_1 - y_2$  and  $u_2 - y_1$  and the controllers can be designed independently if the input-output pairs are reordered.

- Otherwise, the pairs are not decoupled and the decentralized controller should be designed carefully. For stable plants, input-output pairs with negative steady state RGA elements should be avoided. Otherwise, if the sub-controllers are designed independently each with integral action, then the interactions will cause instability either when all the loops are closed or when the loop corresponding to the negative relative gain becomes inactive.

Notice that the RGA depends on the frequency. Therefore, the previous analysis should be done in the frequency range of interest; i.e. the range where LFOs arise.

#### 3.3.2. MSF

It is the building block for Individual Channel Analysis and Design (ICAD) [29, 30]. An appropriate interpretation of the MSF at low and high frequency allows to determine: the existence and required structure of diagonal controllers, the dynamical structure of the closed-loop system, a reliable measurement of robustness, and the possibility to satisfy design specifications [18].

In ICAD, the dynamical structure of  $G(s)$  is determined by individual channels  $C_i(s)$  resulting from pairing each input to each output by means of diagonal controllers [29]. Consider the  $2 \times 2$  system in (12) and the decentralized controller (15). Then the closed-loop system can be represented as two SISO individual channels  $C_i(s)$ , each including a feedback loop and its controller, as shown in Fig. 2. The multivariable structure of the plant is encapsulated by the scalar transfer function  $\gamma_a(s)$ . This representation is equivalent to the original system with no loss of information [29].

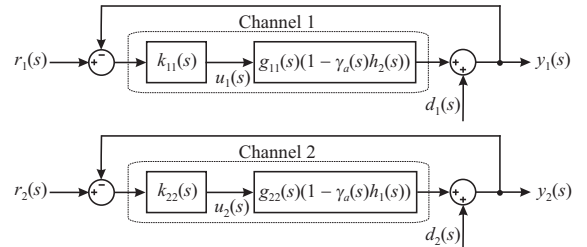


Figure 2: Individual channel representation of a  $2 \times 2$  system

In general,  $C_i(s)$  has the open loop SISO transmittance [29]

$$C_i(s) = k_{ii}(s)g_{ii}(s)(1 - \gamma_a(s)h_j(s)) \quad (16)$$

where

$$\gamma_a(s) = \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)} \quad (17)$$

is the MSF and

$$h_j(s) = \frac{k_{jj}(s)g_{jj}(s)}{1 + k_{jj}(s)g_{jj}(s)} \quad (18)$$

describes the impact of controller  $k_{jj}(s)$  on the  $i$ -th control loop, subjected to the disturbances

$$d_i = \frac{g_{ij}(s)}{g_{jj}(s)}h_j(s)r_j(s), \quad (19)$$

which represent the effect of reference  $r_j(s)$  on channel  $C_i(s)$ . As it can be seen from Fig. 2 and (16)-(19), the behavior of  $C_i(s)$  is both affected by its controller  $k_{ii}(s)$  and by channel  $C_j(s)$  [29, 30]. Moreover, the MSF [18]:

- determines the dynamical characteristics of each input-output configurations and indicates the potential performance of a feedback control system,
- its adequate interpretation ensures an effective control system design,
- its magnitude quantifies the coupling between input-output channels in the frequency domain,
- is related to the plant transmission zeros (zeros of  $1-\gamma_a(s)$ ,  $|G(s)| = g_{11}(s)g_{22}(s) - g_{12}(s)g_{21} = 0$ ).
- $\gamma_a(s) = 1$  determines the non-minimum phase condition,
- its closeness to (1,0) in the Nyquist plot indicates to what extent the plant is sensitive to uncertainty in terms of gain and phase margins.

If the channels are defined by pair  $u_1$ - $y_2$  and  $u_2$ - $y_1$ , then the MSF is given by

$$\gamma_b(s) = \frac{g_{11}(s) \cdot g_{22}(s)}{g_{12}(s) \cdot g_{21}(s)} = \gamma_a^{-1}(s) \quad (20)$$

and

$$C_i(s) = k_{ij}(s)g_{ji}(s) (1 - \gamma_b(s)h_j(s)). \quad (21)$$

Notice that  $H$  can be expressed in terms of the MSFs [18]:

$$H = \begin{bmatrix} \frac{1}{1-\gamma_a(s)} & \frac{\gamma_a(s)}{\gamma_a(s)-1} \\ \frac{\gamma_a(s)}{\gamma_a(s)-1} & \frac{1}{1-\gamma_a(s)} \end{bmatrix} = \begin{bmatrix} \frac{\gamma_b(s)}{\gamma_b(s)-1} & \frac{1}{1-\gamma_b(s)} \\ \frac{1}{1-\gamma_b(s)} & \frac{\gamma_b(s)}{\gamma_b(s)-1} \end{bmatrix}. \quad (22)$$

## 4. Case Study

The different methods presented in Section 3 are assessed through the test system shown in Fig. 3, which has been previously used to show the interaction between conventional generation and WPPs [31]. The WPP including converter-based wind turbines is assumed as a negative load, since the converter dynamics are considerably faster than the electromechanical dynamics that this work is focused on. For the same reason, the transmission lines are modeled as reactances, with the capacitor dynamics in PI representations being neglected. This approach has been previously used in [32]. The system parameters can be found in the Appendix.

The linearized system is represented in state-space form as (2), where

$$x = [\delta \ \omega \ E']^T, \quad u = [P_{wt} \ Q_{wt}]^T, \quad y = [V_{wt} \ \theta_{wt}]^T \quad (23)$$

The state variables  $x$  represent the angle and frequency of the rotor and the exciter control variable of the synchronous machine. The input  $u$  is the aggregated active

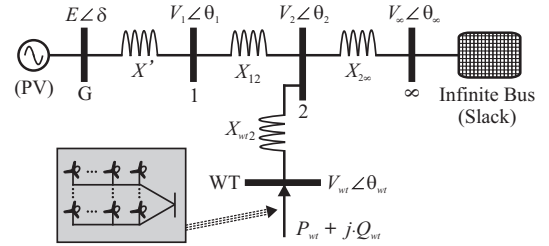


Figure 3: Electrical network representation of the power system under study

and reactive power and the output  $y$  is the voltage magnitude and phase angle of the WPP. Matrix  $A$  has a lightly damped oscillation mode at  $\lambda_{12} = -0.0119 \pm 8.30375j$ , with frequency 1.322 Hz and a damping coefficient of 0.00144.

As it can be seen in (23), the system is  $2 \times 2$ . Although simple in nature, it is representative enough to clearly assess the tools presented in Section 3. The use of more complex system representation is out of the scope of this work; however, the tools here presented could be equally applied to systems with additional inputs and outputs. It should also be noted that the impact of measuring remote signals using wide-area measurement systems could be included through the introduction of time delays, but it is out of scope of this paper. Instead, it is assumed that all signals are available (either through direct measurement or estimation).

### 4.1. Controllability and Observability Measures Comparison

The controllability and observability measures previously presented were compared using the system under study. According to (23), four possible signal pairs can be analyzed:  $P_{wt}$ - $V_{wt}$ ,  $Q_{wt}$ - $V_{wt}$ ,  $P_{wt}$ - $\theta_{wt}$  and  $Q_{wt}$ - $\theta_{wt}$ .

#### 4.1.1. PBH test

The singular values corresponding to the  $\mathcal{C}(\lambda_1)$  and  $\mathcal{O}(\lambda_1)$  are listed in Tables 1 and 2. Notice that matrix  $\mathcal{C}(\lambda_1)$  depends on matrices  $b_i$  and not on  $c_j$ . For this reason, Table 1 has only two columns. The same applies to  $\mathcal{O}(\lambda_1)$ , with results in Table 2. The largest of the minimum singular values for the possible signal pairs is highlighted. It can be observed that the most controllable pairs correspond to those with the active power delivered by the WPP ( $P_{wt}$ ) as an input. The most observable pairs are those with the phase angle of the WPP connection point ( $\theta_{wt}$ ) as an output.

$P_{wt}$ - $V_{wt}/P_{wt}$ - $\theta_{wt}$	$Q_{wt}$ - $V_{wt}/Q_{wt}$ - $\theta_{wt}$
78.9950	77.6115
8.3223	8.3224
<b>1.6480</b>	0.5251

Table 1: Singular Values of Matrix  $\mathcal{C}(\lambda_1)$

It can be concluded that the best signal pair is  $P_{wt}$ - $\theta_{wt}$  since it renders the largest of the minimum singular values for both matrices. Also, it can be seen that the worst signal



$P_{wt}-V_{wt}/Q_{wt}-V_{wt}$	$P_{wt}-\theta_{wt}/Q_{wt}-\theta_{wt}$
77.461	77.462
8.329	8.323
0.002	<b>0.045</b>

Table 2: Singular Values of Matrix  $\mathcal{O}(\lambda_1)$

pair is  $Q_{wt}-V_{wt}$ . However, the next best possible pair is not clear from the remaining options since the PBH test does not provide any joint controllability and observability measure.

#### 4.1.2. Residue Analysis

The magnitude of the residues are listed in Table 3. It can be observed that the maximum value corresponds to the pair  $P_{wt}-\theta_{wt}$ . This is consistent with the information provided by the PBH test and this signal pair should be selected as a first option. It should be mentioned that a well-known PSSs design method to damp oscillations exists based on residue analysis [24]. Conversely to the PBH test, it clearly defines the best order of the signal pairs due to its joint controllability and observability measurement test.

	$P_{wt}-V_{wt}$	$Q_{wt}-V_{wt}$	$P_{wt}-\theta_{wt}$	$Q_{wt}-\theta_{wt}$
$ R_i $	0.018	0.006	<b>0.349</b>	0.109

Table 3: Residue values

#### 4.1.3. Geometric Measures

These are given in Table 4. The joint controllability and observability measure  $m_{co}$  shows that the signal pair to be selected must be  $P_{wt}-\theta_{wt}$  since it has the maximum value among the input-output pairs. These results are consistent with those obtained previously. However, in this case the information is clearer since the values are normalized.

	$P_{wt}-V_{wt}$	$Q_{wt}-V_{wt}$	$P_{wt}-\theta_{wt}$	$Q_{wt}-\theta_{wt}$
$m_c$	<b>0.9928</b>	0.9927	<b>0.9928</b>	0.9927
$m_o$	0.3842	0.3842	<b>0.9505</b>	<b>0.9505</b>
$m_{co}$	0.3815	0.3814	<b>0.9437</b>	0.9436

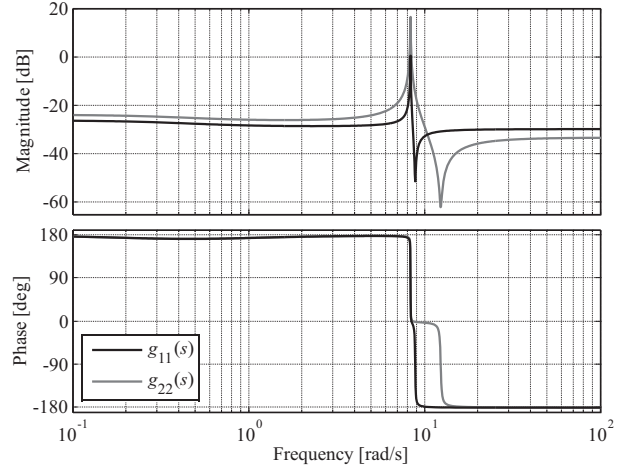
Table 4: Geometric Measures

#### 4.2. RPHZs

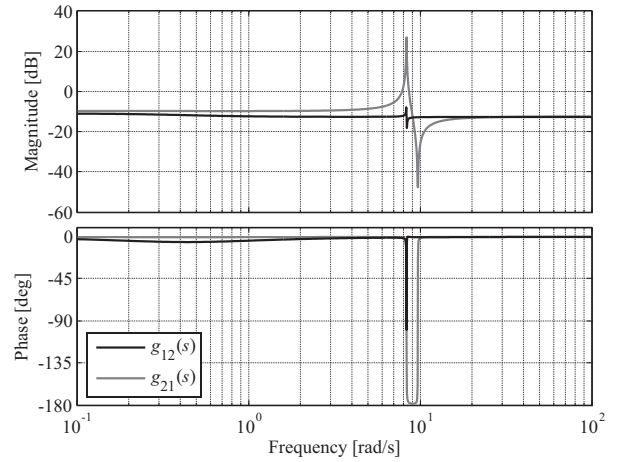
The transfer function corresponding to the system under analysis is given by  $G(s)$  in (12), where

$$\begin{aligned}
g_{11}(s) &= -\frac{0.032727(s+0.5314)(s^2-0.08487s+78.87)}{(s+0.3985)(s^2+0.02391s+68.95)} \\
g_{12}(s) &= \frac{0.23117(s+0.4832)(s^2+0.01688s+69.35)}{(s+0.3985)(s^2+0.02391s+68.95)} \\
g_{21}(s) &= \frac{0.23766(s+0.4036)(s^2+0.0181s+93.34)}{(s+0.3985)(s^2+0.02391s+68.95)} \\
g_{22}(s) &= -\frac{0.021838(s+0.535)(s^2-0.2433s+151.9)}{(s+0.3985)(s^2+0.02391s+68.95)}
\end{aligned} \quad (24)$$

Fig. 4 shows the Bode plots of the individual transfer functions in (24). For clarity, Fig. 4(a) features the diagonal elements  $g_{ii}(s)$  and Fig. 4(b) the off-diagonal entries  $g_{ij}(s)$  of  $G(s)$ . It can be clearly seen that the damping controller should act in the region 8-11 rad/s (1.2-1.75 Hz).



(a)  $g_{11}(s)$  and  $g_{22}(s)$



(b)  $g_{12}(s)$  and  $g_{21}(s)$

Figure 4: Bode diagrams of transfer functions  $g_{ij}(s)$

In equation (24), it can be seen a pair of complex RHPZs in the individual transfer functions  $g_{11}(s)$  and  $g_{22}(s)$  close to the lightly damped mode  $\lambda_{12} = -0.01196 \pm j8.3038$ . In the Bode plots in Fig. 4(a), a resonance peak caused by the low damping of  $\lambda_{12}$  can be observed. The contribution of 180 degrees of the non-minimum phase zeros is also clear in these plots. In the rest of the frequencies, the magnitudes of  $g_{11}(s)$  and  $g_{22}(s)$  are below -20 dB, suggesting a small contribution of input  $P_{wt}$  to output  $V_{wt}$  and input  $Q_{wt}$  to output  $\theta_{wt}$ . Fig. 4(b) shows  $g_{12}(s)$  and  $g_{21}(s)$ . The proximity of the oscillation mode  $\lambda_{12}$  to the lightly damped zeros tends to cancel the resonances peaks; however, in these cases the minimum phase zeros contribute with positive phase. The magnitude of transfer functions  $g_{ij}(s)$  is higher compared to that of  $g_{ii}(s)$ .

If disturbances at the voltage are considered, the closed-loop transfer is given by

$$g_{cl,ij}(s) = (1 + K_{ij}(s)g_{ij}(s))^{-1}.$$

Due to the RHPZs in  $g_{11}$  and  $g_{22}$  ( $\xi_{11} = 0.0424 \pm 8.8808j$

and  $\xi_{22} = 0.1216 \pm 12.3242j$ ), regardless the controller

$$g_{cl,11}(\xi_{11}) = g_{cl,22}(\xi_{22}) = 1.$$

This implies that it will be difficult to reduce the resonance peak and thus to increase the damping of mode  $\lambda_{12}$ , especially in case of  $g_{11}$  where  $\xi_{11}$  is quite close to  $\lambda_{12}$  [26].

This preliminary analysis suggests avoiding signal pairs directly associated to individual elements  $g_{11}(s)$  and  $g_{22}(s)$  due to the limitations imposed by the presence of RHPZs; i.e.,  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$ . Nevertheless, since no conclusion has been made about the internal coupling of the plant it is not apparent how the RHPZs of  $g_{11}(s)$  and  $g_{22}(s)$  will reflect on the signal pairs associated to off-diagonal transfer functions  $g_{12}(s)$  and  $g_{21}(s)$  (i.e.,  $P_{wt}-\theta_{wt}$  and  $Q_{wt}-V_{wt}$ ).

### 4.3. Input-output interactions

#### 4.3.1. RGA

The elements of matrix  $H$  for the  $2 \times 2$  system under study are

$$\begin{aligned} \eta_{11} &= \frac{-0.0132(s+0.54)(s+0.53)(s^2-0.085s+77.87)(s^2-0.24s+154.9)}{(s+0.484)(s+0.398)(s^2+0.0239s+68.95)(s^2+0.018s+92.85)} \\ \eta_{12} &= \frac{1.013(s+0.48)(s+0.4)(s^2+0.017s+69.35)(s^2+0.018s+93.34)}{(s+0.484)(s+0.398)(s^2+0.0239s+68.95)(s^2+0.018s+92.85)} \end{aligned} \quad (25)$$

Fig. 5 shows the Bode diagrams of the diagonal and off-diagonal RGA entries. It can be seen that  $\eta_{11}(s)$  (diagonal elements of  $H$ ) are negative for most frequencies and their magnitude small. This can be concluded after examination of (25) and the phase plot in Fig. 5. Phases of 180 deg and  $-540$  deg correspond to negative magnitudes, and overall  $\eta_{11}(s)$  will only be positive for a narrow margin of frequencies between 8 – 10 rad/s. Conversely, the magnitude of  $\eta_{12}(s)$  (off-diagonal elements of  $H$ ) is close to unity (i.e., 0 dB), as shown by Fig. 5. Simple inspection of (25) shows that  $\eta_{12}(s)$  has a positive magnitude for all frequencies. This is corroborated by the nearly constant phase of 0 deg in Fig. 5.

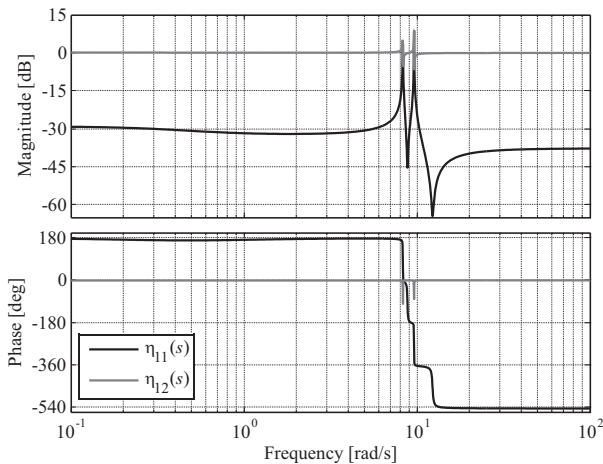


Figure 5: Bode diagrams of RGA matrix  $H$  entries

As outlined in Section 3.3, in case of using decentralized controllers with independently designed elements, input-output pairs with positive diagonal (or off-diagonal) entries are recommended. Since the magnitude of  $\eta_{11}(s)$

is negative for most frequencies, pairs  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$  should be avoided. Furthermore, since the magnitude of  $\eta_{12}$  is close to unity, the corresponding input-output pairs (i.e.,  $P_{wt}-\theta_{wt}$  and  $Q_{wt}-V_{wt}$ ) could be treated as SISO plants. It should be noted that the change of sign at some frequencies in  $\eta_{11}(s)$  implies that RHPZs are present in plant  $G(s)$ , which is consistent with (24). However, it is not clear how those non-minimum phase zeros will affect the control system design and performance arising from the pair definition.

#### 4.3.2. MSF

Using (17) and (20), the MSFs of (24) become

$$\begin{aligned} \gamma_a &= \frac{76.87(s+0.483)(s+0.404)(s^2+0.017s+69.35)(s^2+0.018s+93.34)}{(s+0.535)(s+0.531)(s^2-0.0849s+77.87)(s^2-0.243s+151.9)} \\ \gamma_b &= \frac{0.013(s+0.535)(s+0.531)(s^2-0.085s+77.87)(s^2-0.24s+151.9)}{(s+0.483)(s+0.404)(s^2+0.0169s+69.35)(s^2+0.0183s+93.34)} \end{aligned} \quad (26)$$

An adequate interpretation of the MSFs allows a complete evaluation of the control system design limitations arising from the presence of RHPZs and the multivariable character of the plant. This goes beyond the information provided by the RGA analysis [18]. Since  $G(s)$  was obtained from a state-space form, the number of RHPZs of  $(1 - \gamma_a(s))$  is given by [20]

$$Z = N + P - Q, \quad (27)$$

where  $Z$  is the number of RHPZs of  $(1 - \gamma_a(s))$ ,  $P$  is the number of right hand plane poles (RHPPs) of  $\gamma_a(s)$ ,  $N$  is the number of clockwise encirclements to  $(1, 0)$  in the Nyquist plot of  $\gamma_a(s)$ , and  $Q$  is the number of eigenvalues in the right hand plane of the state-space representation. This is an application of the Nyquist stability criterion [20].

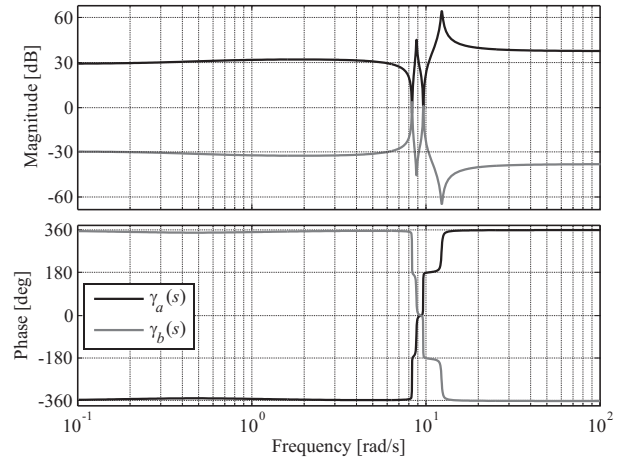


Figure 6: Assessment of MSFs  $\gamma_a(s)$  and  $\gamma_b(s)$

Fig. 6 shows the Bode plots of MSFs  $\gamma_a(s)$  and  $\gamma_b(s)$ . It can be seen that the coupling resulting from pairing  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$  is high for all frequencies, as evidenced by the magnitude of  $\gamma_a(s)$  above 20 dB. From (26), it is evident that  $\gamma_a(s)$  has four RHPPs; i.e.,  $P = 4$ . As shown by (24), for this system  $Q = 0$ . Since  $\gamma_a(0) = 28.85$ , the



Nyquist plot starts at the right side of  $(1, 0)$ . The Nyquist plot of  $\gamma_a(s)$  given in (26) encircles  $(1, 0)$  four times in counter-clockwise direction, implying that  $N = -4$ . Thus, applying (27)  $(1 - \gamma_a(s))$  contains no RHPZs; i.e.,  $Z = 0$ .

The information obtained from the previous analysis is revealing. When considering the signal pairs  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$ , the associated individual channels defined by (16) are non-minimum phase. This is not a consequence of the multivariable character of the plant, as evidenced by the lack of RHPZs in  $(1 - \gamma_a(s))$ , but to those RHPZs appearing in the diagonal transfer functions  $g_{11}(s)$  and  $g_{22}(s)$  (equation (24)). Examination of the Bode plots  $g_{ii}(s)$  in Fig. 4 shows that the frequency of those RHPZs is around the frequency of the oscillation mode  $\lambda_1$ . In order to avoid instability, the bandwidth of the oscillation damper should be restricted below the range of frequencies at which it should act to damp electromechanical oscillations. These are two conflicting and irreconcilable control design objectives and, therefore, the MIMO damper design employing signal pairs  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$  is not recommended. Even if this was not the case, the fact that  $\gamma_a(s)$  has RHPPs implies that  $(1 - \gamma_a(s))$  will preserve such structure. Thus, the damping controller would require the stabilization of an unstable plant with non-minimum phase zeros, which is not a trivial task [21].

Fig. 6 also shows that the internal coupling of the plant arising from pair  $P_{wt}-\theta_{wt}$  and  $Q_{wt}-V_{wt}$  is weak: the magnitude of  $\gamma_b(s)$  is below  $-20$  dB for most frequencies except for those where the damping controller should act. From (26) it is evident that  $\gamma_b(s)$  has no RHPPs ( $P = 0$ ). The Nyquist trajectory of  $\gamma_b(s)$  given by (26) starts to the left of  $(1, 0)$  and no encirclements to this point occur, implying that  $N = 0$ . Applying (27),  $Z = 0$  and thus  $(1 - \gamma_b(s))$  contains no RHPZs. Moreover, transfer functions  $g_{12}(s)$  and  $g_{21}(s)$  have no RHPZs. In spite of the presence of RHPZs in  $g_{11}(s)$  and  $g_{22}(s)$ , this does not reflect on the input-output channel definition associated to  $\gamma_b(s)$  in (16). Based on this analysis, the input-output pairs  $P_{wt}-\theta_{wt}$  and  $Q_{wt}-V_{wt}$  should be considered for a MIMO controller design.

#### 4.4. Simulations

In order to illustrate the previous results, time domain simulations were carried out. The test system of Fig. 3 was implemented in MATLAB/Simulink. A controller of the form  $K_{pss} = K(sT_w/(sT_w + 1))$  was designed using a root locus approach, where  $K_{pss}$  was formed by a washout filter with a time constant  $T_w = 2$ s, a gain  $K = 3$  and a limiter. Such a controller structure was used for the case of independent four input-output SISO pairs and when evaluating MIMO configurations. The controller parameters remained the same to offer a meaningful comparison while ensuring system stability for all SISO cases. In other words, for the following results  $K_{pss} = k_{11} = k_{22} = k_{12} = k_{21}$ .

Fig. 7 shows the simulation results (input and output variables) for SISO designs considering input-output pairs  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$  and for the case of a decentralized MIMO design considering the pairs defined by  $\gamma_a(s)$  (i.e.,

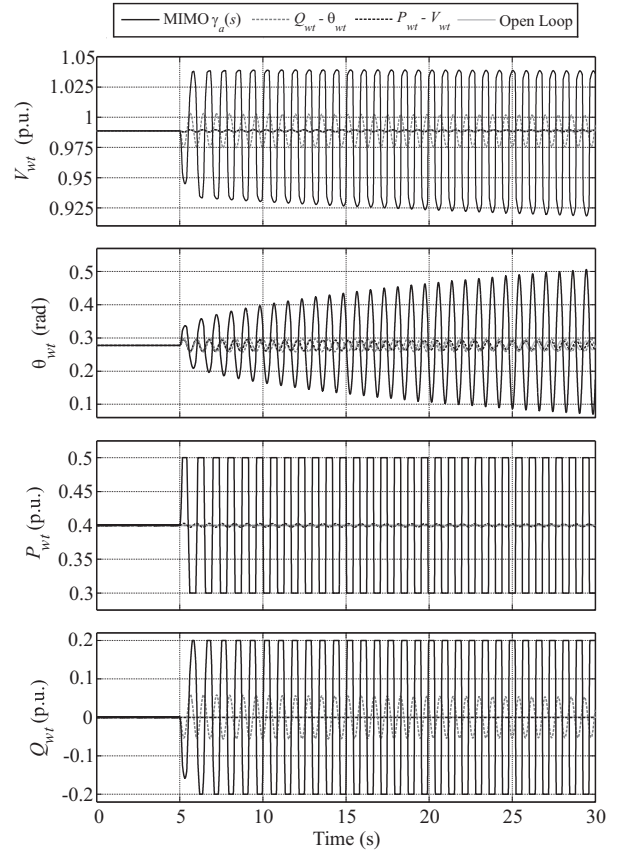


Figure 7: Simulation responses (voltage magnitude, phase angle of the voltage, active power, and reactive power at WPP bus) for SISO and MIMO designs involving input-output pairs  $P_{wt}-V_{wt}$  and  $Q_{wt}-\theta_{wt}$ .

$C_1(s): P_{wt}-V_{wt}$  and  $C_2(s): Q_{wt}-\theta_{wt}$ ). The simulation starts with the system at steady state. At 5s, an impulse variation on the mechanical power reference of the synchronous machine is done, in order to excite the oscillation mode existing on the system. As it can be observed, the system becomes unstable for the MIMO design. This was expected since MSF  $\gamma_a(s)$  features RHPPs and the simple controllers used are not able to modify its structure. It could be argued that a more complex MIMO controller may stabilize the plant, but this is out of the scope of this work. Conversely, the system is stable when considering the designs for the SISO pairs, but non-minimum phase zeros are present. Moreover, the damping contribution is weak in either SISO case and the oscillations are not damped during the simulation horizon. It could be argued that the performance may be improved by increasing the proportional gain of the controller, but care should be exercised since the system may become unstable due to the presence of RHPZs.

Fig. 8 shows the simulation results for the SISO designs considering input-output pairs  $P_{wt}-\theta_{wt}$  and  $Q_{wt}-V_{wt}$  and for the case of a decentralized MIMO design considering the pairs defined by  $\gamma_b(s)$  (i.e.,  $C_1(s): P_{wt}-\theta_{wt}$  and  $C_2(s): Q_{wt}-V_{wt}$ ). As it can be seen, in both SISO cases the oscillations are damped after a transient period. When consider-

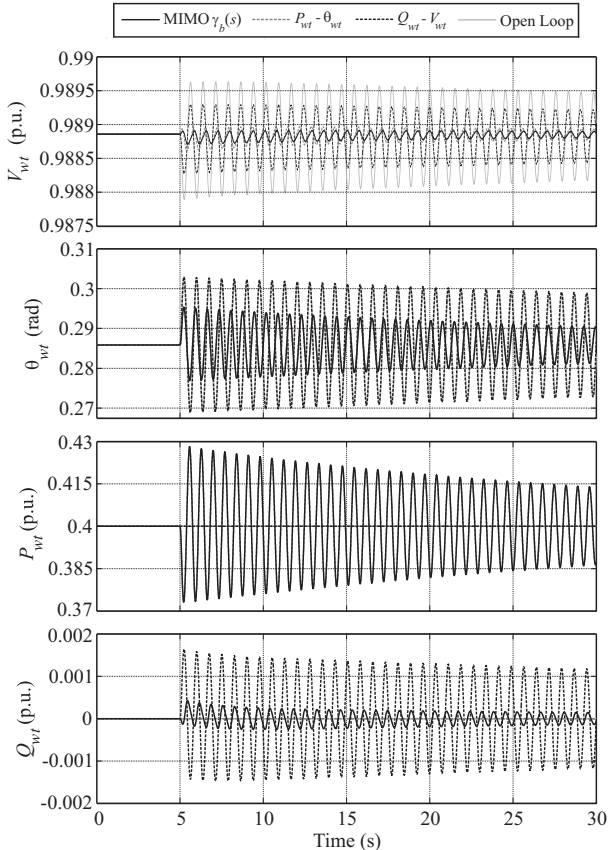


Figure 8: Simulation responses (voltage magnitude, phase angle of the voltage, active power, and reactive power at WPP bus) for SISO and MIMO designs involving input-output pairs  $P_{wt}-\theta_{wt}$  and  $Q_{wt}-V_{wt}$ .

ing the SISO pair  $P_{wt}-\theta_{wt}$ , the magnitude of the voltage, its phase angle and the reactive power show less oscillations than for the SISO case with  $Q_{wt}-V_{wt}$ . This was expected from the analysis carried out in previous sections. When using the same controller in both individual channels, it can be seen that the MIMO design produces the best damping performance in all variables except for the case of active power, where the oscillations take a longer time to be eliminated. This superior performance was expected since the MIMO design takes the damping contribution of both feedback loops.

It should be emphasized that the results obtained in Figs. 7 and 8 agree on well with the analyses carried out with the controllability and observability tools and the frequency domain approaches from the previous sections. It is also apparent that the best performance for any possible SISO alternative is achieved when considering input-output pair  $P_{wt}-\theta_{wt}$ .

## 5. Conclusion

Several tools have been used to decide which input-output configuration is more adequate to damp oscillations in power systems with WPPs. Controllability and observability tests have been analyzed – in particular, those al-

lowing the evaluation of specific oscillation modes (PBH, residues and geometric measures). Among them, the most recommended are the geometric measures since they provide a normalized joint controllability and observability measure. The normalization is useful for the input-output selection because in general the input-output pairs have different scales.

The limitations imposed by the presence of RHPZs on performance have been considered. The stabilization of a plant with RHPZs gives rise to fixed points in the frequency response. As a consequence, if the RHPZs are close to the oscillation modes, there is no controller capable of achieving a substantial increment in damping.

Interactions among input-output pairs were analyzed to assess the possibility of a multivariable control design. RGA and MSF are useful tools to determine the most adequate signal pairs for decentralized control. An advantage of using the MSF approach over the RGA analysis, in addition to the definition of signal pairs afforded through both approaches, is that the potential dynamic performance and control system design of the oscillation damper can be evaluated. Moreover, through a careful analysis of the MSFs a clear effect of the limitations arising from RHPZs can be clearly defined.

In order to illustrate the different alternatives, a power system including a WPP was analyzed. A simple oscillation damper obtained through classic control design was employed to carry out time domain simulations. Simulation results show a system behavior in agreement with the previous analyses.

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## Appendix

The power system, synchronous generator and operation point values are presented as follows

$P_g$	$P_{wt}$	$Q_{wt}$	$E'$	$\omega_s$ (rad/s)	$X_{12}$	$X_{wt2}$	$X_{2\infty}$
0.8	0.4	0	1	$100\pi$	0.1	0.1	0.2

Table .5: Power system parameters (in p.u. except indicated)[31]

$T_0$	$H$ (s)	$X_{ta}$	$X'$
6	4	1.1	$0.15 + X_{ta}$

Table .6: Synchronous machine parameters (in p.u. except indicated)[31]

$E'_0$	$\delta_0$	$V_{wt0}$	$\theta_{wt0}$	$V_{20}$
1	0.5283	0.9889	0.2858	0.9897
$\theta_{20}$	$V_{10}$	$\theta_{10}$	$V_{ib0}$	$\theta_{ib0}$
0.2449	0.9961	0.3262	1	0

Table 7: Operating point, where voltages and angles are in expressed in p.u and in rad, respectively.

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