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Optimal gain-scheduled control of fixed-speed active stall wind turbines *

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Abstract

For a smooth integration of large wind farms into the utility grids, the individual wind turbines must be able to achieve various power control objectives. In this context, we focus our attention on the control of fixed–speed active stall wind turbines. This sort of turbine includes a pitch servomechanism to induce stall on the blades, thereby having control on the output power. In this paper we develop a methodology to design optimal gain–scheduled pitch controllers valid for the whole operating region of the wind turbine. The proposed solution uses concepts of linear parameter–varying system theory. In addition to providing a formal framework for the control design, this theory gives guarantees of stability and performance. Further, because of the similarities with $\mathcal{H}_\infty$ control, the tools developed for the controller design are very familiar to the control community. The main features of the proposed controller are assessed by means of numerical simulations obtained for realistic wind speed profiles and power production demands.

Keywords Wind energy; power control; gain scheduling; linear parameter–varying systems.

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1 Introduction

Traditionally, the primary objective of large wind turbine control systems has been to reduce the cost per kWh. This means the maximisation of the power production, only limited by the rated power of the machine. At wind farm level, the control decisions have been limited for a long time to shut down and start up the wind turbines. However, with the operational capacity of modern wind farms being comparable to conventional power plants and the penetration of wind energy into the electricity markets continuously rising, the technical specifications for wind power grid connection are becoming more demanding [1, 2, 3, 4]. Therefore, there exists at present a growing interest in reducing the impact of wind energy on power system quality and reliability. Moreover, modern wind farms are expected to share some of the duties currently performed by conventional power plants such as voltage and frequency regulation, dynamic stability, etc. [2, 5, 6]. To this end, wind farms must exhibit flexibility for the control of the active power supplied to the grid as well as of the reactive power absorbed from or injected to the grid [7].

Currently, the dominant idea is to control the active and reactive power production by means of a two–level control system, namely a centralised wind farm control level and a local wind turbine control level. The aim of the centralised controller is for the wind farm to comply with the orders of the power system operator [8, 9]. It must estimate the wind power available in the farm and compute on–line power reference signals for each wind turbine. On the other hand, the wind turbine controller must track its power references. Hence, the regulation of wind farms translates into new specifications for wind turbines operation. For instance, wind turbines can be required to regulate power at a constant value lower than rated power (balance control), to limit power production in such a way that a specified power reserve is always available (delta control), to limit the rate of change of the generated power (gradient control), etc. [2, 8]. These new specifications enlarge the conventional operating locus primarily determined to maximise the energy capture [10, 11].

The dynamic behaviour of wind turbines strongly varies along their operating range. Therefore, gain scheduling techniques, which are specifically developed to cope with non-linear dynamics using tools and concepts of linear systems, have been widely applied to the design of wind turbine control systems [12, 13, 14, 15]. The gain–scheduling problem can be formally formulated in the context of linear parameter–varying (LPV) systems

[16, 17]. In addition to accomplishing guarantees of stability and performance, this approach simplifies considerably the control design. Further, because of the similarities with $\mathcal{H}_\infty$ control, the new tools to design LPV gain–scheduled controllers are very intuitive and familiar to the control community. Moreover, parameter and model uncertainties can be dealt with in a straightforward fashion. In previous works, LPV controllers have been developed for variable–speed wind turbines operating along the conventional control locus (i.e. along the curve of maximum power capture up to rated power) [18, 19, 20, 21, 22]. In the current paper we develop an LPV controller showing optimum performance along an operating range enlarged to cope with the new specifications for the fixed–speed active–stall configuration. Naturally, the expansion of the operating locus associated to the new control objectives brings with it a wider variety of dynamic behaviours. This strengthens the need for controllers capable of adapting themselves to the dynamic changes experienced by the turbine.

The paper is organised as follows. In the next section, the turbine control strategy is discussed. Then, in section 3, the procedure to design an optimal LPV gain–scheduled controller for the pitch servo system is presented. In section 4, the performance of the proposed controller is evaluated by simulation under realistic wind speed and power reference profiles. Finally, the last section summarises the conclusions of the work.

2 Control strategy

The active stall control method is a popular alternative to pitch regulation in the medium to high power scale. Without the inherent complexity of variable–speed operation but at the cost of higher mechanical stresses, it achieves a smoother power regulation at high wind speeds. An active stall wind turbine comprises a variable–pitch wind rotor, with the pitch angle being controlled to induce stall on the blades. The pitch mechanism allows a flat power regulation characteristic above rated wind speed, whereas power efficiency at low wind speeds can be improved to a certain extend. This variable–pitch wind rotor usually drives a squirrel cage induction generator directly coupled to the AC grid (Fig. 1). That is, the turbine works at fixed–speed except for the small slip inherent to the operation of the induction machine. There also exists a double–speed version, where the synchronous speed can be switched between two fixed values by reconnecting the stator windings of the machine. Double speed operation gives improved conversion efficiency and lower noise at low wind speeds. Finally, there is a capacitor bank with the purpose of having some control of reactive power.

The power P_r captured by a wind rotor of diameter $2R$ facing an airflow of speed V and density ρ is

$$P_r = \frac{1}{2}\pi R^2 \rho C_P(\lambda, \beta) V^3, \tag{1}$$

where C_P describes the turbine aerodynamics. This power coefficient is usually written as function of the pitch angle β and the tip–speed–ratio $\lambda = R\Omega_r/V$, with Ω_r being the rotational speed that is almost constant in the current configuration. Coefficient C_P takes its maximum value $C_{P_{\max}}$ at an optimum tip–speed–ratio λ_{o} and β_{o} ($C_{P_{\max}} = C_P(\lambda_{\text{o}}, \beta_{\text{o}})$). This means that a fixed–speed wind turbine is optimally loaded only at one wind speed. For any other wind speed, sub–optimum operation can be achieved by pitching the blades a small angle in positive or negative direction around β_{o}.

Fig. 2 shows the conventional control strategy of a fixed–speed active stall wind turbine. In the top part of the figure, the control strategy is plotted on the power – wind speed plane, whereas the bottom part of the figure depicts the corresponding power conversion efficiency (C_P) as function of wind speed. This conventional control strategy basically consists in capturing as much power as possible between cut–in ($V_{\min}$) and cut–off ($V_{\max}$) wind speeds without overloading the turbine, i.e. without exceeding rated power. This strategy can be divided into two regions having different control goals:

$\beta-$**optimisation mode** Below rated wind speed, the power the turbine can produce is lower than rated. Therefore, the conventional control objective in low wind speeds consists in capturing as much power as possible. As explained before, the coefficient C_P is maximised as much as possible by adjusting the pitch angle as function of wind speed. This task is performed in open–loop. That is, after averaging the wind speed over a certain period of time, the controller looks up the appropriate value of β in a look–up table [23]. When building this table, it should be kept in mind that the $C_P - \beta$ curves have sharp maxima in low wind speeds whereas they are flat in high wind speeds. Therefore, the table must contain several points in the low wind speed range to meet the optimum β accurately [24].

Limitation mode Above rated wind speed, the power available in the wind exceeds rated power. Therefore, the control strategy in high wind speeds consists in regulating the turbine at rated power, thereby avoiding overloads. This task is done in closed–loop. The stall effect is controlled by pitching the blades. Thus, properly adjusting the pitch angle as function of the output power error, a flat power regulation is achieved.

When the wind turbine is part of a large–scale wind farm, the control strategy may differ from the conventional one consisting in maximizing the power production without exceeding rated power. For instance, it may be required to regulate the turbine at a power set–point below rated power or to track a time–varying power reference signal specified by the centralised farm controller [25]. Now, the power optimisation and power limitation modes of operation are not only determined by the current wind speed but also by the power demand.

Fig. 3 shows the operating points of the turbine plotted onto the C_P curve for any feasible control strategy. The point N represents turbine operation at rated wind speed and rated power. This is the nominal point of operation. The line IN represents the $\beta-$optimisation mode of operation. In fact, on this line, the C_P coefficient takes its maximum achievable value for each wind speed between cut–in (I) and rated (N). On the line NO the turbine operates at rated power between rated wind speed (N) and cut–off wind speed (O). Therefore, the line identified by the points INO represents the conventional control strategy. Now, if the power reference may take values between 0% and 100% of rated, then all feasible points of operation covers the whole area delimited by the lines IJ, JQ, IN, NO and OQ. The lines IJ and OQ are the operating loci (from 100% to 0% of rated power) at cut–in and cut–off wind speeds, respectively. Finally, the line that joins the points J and Q is characterised by zero power production. It is clear that the turbine operates in the $\beta-$optimisation mode just on the line IN whereas it operates in the limitation mode on the rest of the region.

3 Linear parameter–varying gain–scheduled controller

In classical gain scheduling techniques, the nonlinear or time–varying plant is linearised around a finite set of operating points and a linear controller is subsequently designed for each of these linear time–invariant plants. Then, the gain–scheduled controller is obtained from the family of linear controllers by means of a switching or interpolation algorithm. Gain scheduling techniques have been extensively used in a wide range of applications. However, in the absence of theoretical foundations, stability, robustness and performance properties of the gain–scheduled controlled system cannot be assessed from the properties of the linear controllers family.

In the early 1990s, Shamma and Athans introduced the linear parameter–varying systems [16]. LPV models are generally obtained by reformulating a nonlinear or time–varying system as a linear system whose dynamics depend on some time–varying exogenous parameters. In addition to providing a formal framework, the concepts of LPV systems simplify the synthesis of gain–scheduled controllers, which can be formulated as a convex optimisation problem with LMIs [26, 17]. The existence of efficient numerical algorithms makes this optimisation approach very effective to solve a wide range of control problems [27]. In LMIs-based LPV gain scheduling techniques, the controller is treated as a unique entity, thereby simplifying the scheduling algorithm. In many aspects, the controller design follows a procedure similar to $\mathcal{H}_\infty$ control, with the difference that the resultant controller is now dependent on the scheduling parameters.

In this section, we present first the derivation of an LPV model for the wind turbine. Then, we proceed to describe the controller setup. Finally, we provide the tools to synthesise the LPV gain–scheduled controller.

3.1 Linear parameter–varying model

The model of a wind turbine comprises several subsystems. Namely, the drive–train and tower dynamics, the aerodynamics, the pitch servo and the generator. The dominant dynamics usually lie in the mechanical subsystem. Therefore, wind turbines are commonly modelled as flexible structures undergoing exogenous torques from the wind and generator.

Drive–train model Pitch angle control has a direct impact on the aerodynamic forces developed on the rotor. Consequently, inappropriate controllers may induce tower bending and vibrations. Fortunately, the structure dynamics can be disregarded during the controller design process since it is outside the control loop [18]. On the contrary, a complete model, i.e., a model including the structure dynamics, must be considered for the proper assessment of controller performance. Therefore, just the first vibration mode of the drive–train will be considered here for the formulation of the LPV model, whereas the simulation results presented later on were obtained using a more complete model of the mechanical subsystem.

The dominant dynamics of the drive–train is modelled as

$$\begin{bmatrix} \dot{\theta}_s \\ \dot{\Omega}_r \\ \dot{\Omega}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\dfrac{K_s}{J_r} & -\dfrac{B_s}{J_r} & \dfrac{B_s}{J_r} \\ \dfrac{K_s}{J_g} & \dfrac{B_s}{J_g} & -\dfrac{B_s}{J_g} \end{bmatrix} \begin{bmatrix} \theta_s \\ \Omega_r \\ \Omega_g \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{T_r}{J_r} \\ -\dfrac{T_g}{J_g} \end{bmatrix} \tag{2}$$

where T_r and T_g are the exogenous torques, namely the aerodynamic torque developed on the wind rotor and the reaction torque of the generator, respectively. The variables and parameters of the model are listed in Table 1.

Aerodynamic model The aerodynamic torque T_r is a nonlinear function of wind speed, rotor speed and pitch angle. It is usually expressed in the form

$$T_r = \frac{1}{2}\rho\pi R^3\, C_T(\lambda, \beta) V^2, \tag{3}$$

where $C_T(\lambda, \beta) = C_P(\lambda, \beta)/\lambda$ is the torque coefficient of the wind rotor. In order to derive an LPV model, (3) is linearised as follows [1]:

$$\hat{T}_r = -B_r(\bar{V}, \bar{\beta}) \cdot \hat{\Omega}_r + k_v(\bar{V}, \bar{\beta}) \cdot v + k_\beta(\bar{V}, \bar{\beta}) \cdot \hat{\beta}, \tag{4}$$

where the bar ($\bar{}$) and hat ($\hat{}$) over the variables mean 'value at the operating point' and 'deviation from the operating point', respectively. The variable v is the turbulence component of the wind V, i.e. $v = V - \bar{V}$. The coefficient B_r denotes the aerodynamic damping intrinsic to the wind rotor, whereas the coefficients k_v and k_β are the sensitivity of the aerodynamic torque to wind turbulence and pitch angle deviations, respectively. The expressions for these coefficients as functions of the operating point (o.p.) are

$$\begin{aligned} B_r(\bar{V}, \bar{\beta}) &= \left.\frac{\partial T_r}{\partial \Omega_r}\right|_{\text{o.p.}} = -\frac{\bar{T}_r}{\bar{\Omega}_r}\left(1 - \left.\frac{\partial C_P/\partial\lambda}{C_P/\lambda}\right|_{\text{o.p.}}\right), \\ k_v(\bar{V}, \bar{\beta}) &= \left.\frac{\partial T_r}{\partial V}\right|_{\text{o.p.}} = \frac{\bar{T}_r}{\bar{V}}\left(3 - \left.\frac{\partial C_P/\partial\lambda}{C_P/\lambda}\right|_{\text{o.p.}}\right), \\ k_\beta(\bar{V}, \bar{\beta}) &= \left.\frac{\partial T_r}{\partial \beta}\right|_{\text{o.p.}} = \frac{\bar{T}_r}{\bar{\beta}}\left.\frac{\partial C_P/\partial\beta}{C_P/\beta}\right|_{\text{o.p.}}. \end{aligned} \tag{5}$$

Fig. 4 shows how the intrinsic aerodynamic damping B_r, and the torque sensitivities to wind turbulence (k_v) and pitch (k_β) of a typical active stall wind turbine vary with mean

[1] Note that the operating point, which is given by the intersection of the wind rotor and generator torque—speed characteristics, is completely determined by $\bar{V}$ and $\bar{\beta}$.

wind speed for different constant–power curves in Fig. 3. It is observed that the intrinsic damping takes small positive values in the optimisation mode of operation. However, when the wind turbine switches to its limitation mode of operation, the intrinsic damping falls rapidly until reaching large negative values. This is particularly true for the conventional control strategy (solid line). Note that large negative damping threatens the stability of the system. The gain k_v is positive and increasing in the optimisation mode. This means that the captured power increases with wind speed and, moreover, increases more rapidly as wind speed rises. In the power limitation mode, this gain decreases until becoming negative when the turbine stalls. This gain gives an idea about the impact of turbulence on the output power regulation. Finally, k_β gives a measure of control sensitivity. Naturally, $k_\beta = 0$ during the optimisation mode since β is selected to maximise the aerodynamic torque. This is not a problem because the pitch control loop is open in this operation mode. In the limitation mode, k_β increases with wind speed. This means that less control effort is needed as wind speed rises and, conversely, that there are some controllability problems at the beginning of the limitation mode where k_β takes small values. In fact, in this situation, fast pitch variations are needed to effectively regulate the output power in the presence of turbulence. This poor controllability imposes some restrictions on the achievable performance in this region.

Pitch actuator model The pitch angle of the blades is modified by a nonlinear servo that generally rotates all the blades –or part of them– in unison. In closed loop the pitch actuator can be modelled as a first–order dynamic system with saturation in the amplitude and derivative of the output signal [12]. Fig. 5 shows a schematic diagram of the first–order actuator model. The dynamic behaviour of the pitch actuator operating in its linear region is described by the differential equation

$$\dot{\beta} = \frac{1}{\tau}(\beta_d - \beta), \tag{6}$$

where β and β_d are the actual and desired pitch angles, respectively.

Power regulation may demand fast and large corrections of the pitch angle, particularly when the sensitivity k_β is low. Consequently, the bounds on the rate of change and amplitude of the pitch angle may have appreciable effects on the power regulation features. To reduce the risks of fatigue damage and undesirable dynamic behaviours, these limits should not be reached during normal operation of the turbine. This restriction must be taken into account during the control design procedure.

Generator model As mentioned above, we disregard here the dynamics of the electrical subsystem since it is much faster than the drive–train and pitch actuator dynamics. Therefore, we describe the generator behaviour by means of its static torque–speed characteristic, which can be approximated by:

$$T_g = B_g(\Omega_g - \Omega_s). \tag{7}$$

LPV model of the wind turbine Incorporating the pitch actuator dynamics (6) to the drive–train model (2), and replacing T_r and T_g with (4) and (7), the dynamics of the entire wind turbine can be expressed in the form of an LPV model

$$G : \begin{cases} \dot{\mathbf{x}}(t) = \mathcal{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathcal{B}_v(\mathbf{p}(t))v(t) + \mathcal{B}(\mathbf{p}(t))u(t) \\ \psi(t) = \mathcal{C}\mathbf{x}(t) \end{cases} \tag{8}$$

where

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} \hat{\theta}_s & \hat{\Omega}_r & \hat{\Omega}_g & \hat{\beta} \end{bmatrix}^T, \\ u &= \hat{\beta}_d, \\ \psi &= \hat{P}_g, \\ \mathbf{p} &= \begin{bmatrix} \bar{V} & \bar{\beta} \end{bmatrix}^T. \end{aligned}$$

The inputs to the model are the turbulence v, which is regarded as a disturbance, and the control action β_d. The output variable is the electrical power supplied to the grid $P_g = \Omega_s T_g$. The parameter vector $\mathbf{p}$ is the set of variables that define the operating point of the wind turbine.

The matrices of the model are

$$\begin{aligned} \mathcal{A}(\mathbf{p}) &= \begin{bmatrix} 0 & 1 & -1 & 0 \\ -\dfrac{K_s}{J_r} & -\dfrac{B_r(\mathbf{p}) + B_s}{J_r} & \dfrac{B_s}{J_r} & \dfrac{k_\beta(\mathbf{p})}{J_r} \\ \dfrac{K_s}{J_g} & \dfrac{B_s}{J_g} & -\dfrac{B_s + B_g}{J_g} & 0 \\ 0 & 0 & 0 & -\dfrac{1}{\tau} \end{bmatrix}, \\ \mathcal{B}(\mathbf{p}) &= \begin{bmatrix} 0 & 0 & 0 & \dfrac{1}{\tau} \end{bmatrix}^T, \\ \mathcal{B}_v(\mathbf{p}) &= \begin{bmatrix} 0 & \dfrac{k_v(\mathbf{p})}{J_r} & 0 & 0 \end{bmatrix}^T, \\ \mathcal{C} &= \begin{bmatrix} 0 & 0 & B_g\Omega_s & 0 \end{bmatrix}, \end{aligned}$$

where the dependence of $\mathbf{p}$ on time is omitted for brevity.

Finally, to completely characterise the LPV model, the region $\mathcal{P}$ where the parameter $\mathbf{p}$ lives must be specified. This region is obtained from Fig. 3 by projecting the shaded area onto the V–β plane. This region is plotted in Fig. 6.

3.2 Controller setup

The design of LPV controllers resembles H_∞ control of linear systems. The control problem is stated in terms of the minimisation of the induced norm of an input–output operator $T_{zw} : w \to z$ that represents the control objectives. Consequently, the first step of the controller design consists in identifying the input variable w –the so–called disturbance– and a performance output z that usually includes control and controlled variables of the system. Then, weighting functions are selected. They are generally linear transfer functions stressing the performance output at the frequencies of interest. The combination of the open–loop system and weighting functions is called augmented plant.

Fig. 7a sketches the block diagram of the control system whereas Fig. 7b shows the corresponding augmented plant.

The wind turbulence is regarded as the disturbance to the plant, i.e. $w = v$. The performance outputs are obtained by passing the control signal $u = \hat{\beta}_d$ and the controlled variable $\psi = \hat{P}_g$ through the weighting blocks $W_u(s)$ and $W_e(s)M(s)$, respectively. Note that the latter weighting function is factorised into two separate functions $W_e(s)$ and $M(s)$. This is for the augmented plant to satisfy stabilizability conditions [18]. Suitable expressions for the weighting functions are

$$\begin{aligned} W_u(s) &= k_u \frac{s/(0.1\omega_u) + 1}{s/(10\omega_u) + 1}, \\ W_e(s)M(s) &= k_e \frac{s/100 + 1}{s + 0.1(1.01 - p_e)}, \end{aligned} \tag{9}$$

where

$$p_e = \begin{cases} 0 & \text{if} \quad k_\beta < 0.05 \\ 1 & \text{if} \quad k_\beta \geq 0.05 \end{cases}$$

On the one hand, $W_u(s)$ weights the control effort with the aim of penalizing fast pitch angle variations. On the other hand, the matrix $W_e(s)M(s)$ stresses the importance of the low frequency components of the power error. This performance specification is relaxed when the sensitivity k_β is low, i.e. when the system exhibits low controllability.

The augmented plant is an LPV system of the form

$$\tilde{G}: \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}_1(\mathbf{p}(t))w(t) + \mathbf{B}_2 u(t) \\ \mathbf{z}(t) = \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{11}w(t) + \mathbf{D}_{12}u(t) \\ y(t) = \mathbf{C}_2\mathbf{x}(t) \end{cases} \tag{10}$$

where the matrices in (10) are readily obtained from the LPV model of the plant (8) and the weighting functions (9). It is immediate to show that the augmented plant is affine in $B_r(\mathbf{p})$, $k_v(\mathbf{p})$, $k_\beta(\mathbf{p})$ and $p_e(\mathbf{p})$. That is, if we define the basis functions

$$\begin{aligned} \phi_1(\mathbf{p}(t)) &= B_r(\mathbf{p}(t)), \\ \phi_2(\mathbf{p}(t)) &= k_v(\mathbf{p}(t)), \\ \phi_3(\mathbf{p}(t)) &= k_\beta(\mathbf{p}(t)), \\ \phi_4(\mathbf{p}(t)) &= p_e(\mathbf{p}(t)), \end{aligned}$$

then the parameter–dependent matrices $\mathbf{A}(\mathbf{p})$ and $\mathbf{B}_1(p)$ can be expressed in the form

$$\mathbf{A}(\mathbf{p}) = \mathbf{A}_0 + \sum_{i=1}^{4} \mathbf{A}_i\phi_i(\mathbf{p}), \qquad \mathbf{B}_1(\mathbf{p}) = \mathbf{B}_{1,0} + \sum_{i=1}^{4} \mathbf{B}_{1,i}\mathbf{x}_i(\mathbf{p}),$$

where the matrices $\mathbf{A}_i$ and $\mathbf{B}_{1,i}$ are independent of $\mathbf{p}$. This property is used in the following subsection to synthesise the controller.

3.3 Controller synthesis

The controller synthesis consists in finding an LPV controller

$$C: \begin{cases} \dot{\mathbf{x}}_c(t) = \mathbf{A}_c(\mathbf{p}(t))\mathbf{x}_c(t) + \mathbf{B}_c(\mathbf{p}(t))y(t) \\ u(t) = \mathbf{C}_c(\mathbf{p}(t))\mathbf{x}_c(t) + \mathbf{D}_c(\mathbf{p}(t))y(t) \end{cases} \tag{11}$$

such that the closed–loop system is stable and the $\mathcal{L}_2$–norm of $\mathbf{z}(t)$, $\|\mathbf{z}(t)\|_2 =: \int_0^\infty \mathbf{z}(t)^T\mathbf{z}(t)dt < \gamma$ for all inputs $w(t)$ satisfying $\|w(t)\|_2 < 1$.

Therefore, from [17] it can be shown that the controller results from solving the following optimisation problem with LMIs:

minimise γ

subject to the LMIs (12) and (13),

where the meaning of the symbol $\star$ is inferred by symmetry[2]:

[2]There currently exist various broadly available numerical algorithms to efficiently solve this sort of inequality.

$$
\begin{bmatrix}
\dot{\mathbf{X}} + \mathbf{X}\mathbf{A} + \hat{\mathbf{B}}\mathbf{C}_2 + (\star) & \star & \star & \star \\
\hat{\mathbf{A}}^T + \mathbf{A} + \mathbf{B}_2\hat{\mathbf{D}}\mathbf{C}_2 & -\dot{\mathbf{Y}} + \mathbf{A}\mathbf{Y} + \mathbf{B}_2\hat{\mathbf{C}} + (\star) & \star & \star \\
(\mathbf{X}\mathbf{B}_1)^T & \mathbf{B}_1^T & -\gamma \mathbf{I}_{n_w} & \star \\
\mathbf{C}_1 + \mathbf{D}_{12}\hat{\mathbf{D}}\mathbf{C}_2 & \mathbf{C}_1\mathbf{Y} + \mathbf{D}_{12}\hat{\mathbf{C}} & \mathbf{D}_{11} & -\gamma \mathbf{I}_{n_z}
\end{bmatrix} < 0, \tag{12}
$$

$$
\begin{bmatrix}
\mathbf{X} & \mathbf{I} \\
\mathbf{I} & \mathbf{Y}
\end{bmatrix} > 0. \tag{13}
$$

The optimisation variables are the Lyapunov functions $\mathbf{X}(\mathbf{p})$ and $\mathbf{Y}(\mathbf{p})$ and the set of auxiliary controller matrices

$$
\hat{\mathbf{A}}(\mathbf{p}) = \hat{\mathbf{A}}_0 + \sum_{i=1}^{n_\mathbf{p}} \phi_i(\mathbf{p})\hat{\mathbf{A}}_i, \qquad \hat{\mathbf{B}}(\mathbf{p}) = \hat{\mathbf{B}}_0 + \sum_{i=1}^{n_\mathbf{p}} \phi_i(\mathbf{p})\hat{\mathbf{B}}_i,
$$
$$
\hat{\mathbf{C}}(\mathbf{p}) = \hat{\mathbf{C}}_0 + \sum_{i=1}^{n_\mathbf{p}} \phi_i(\mathbf{p})\hat{\mathbf{C}}_i, \qquad \hat{\mathbf{D}}(\mathbf{p}) = \hat{\mathbf{D}}_0 + \sum_{i=1}^{n_\mathbf{p}} \phi_i(\mathbf{p})\hat{\mathbf{D}}_i.
$$

The Lyapunov functions can be searched in the set of parameter–dependent matrix functions, i.e.

$$
\begin{aligned}
&\mathbf{X}(\mathbf{p}) = \mathbf{X}_0 + \sum_{i=1}^{n_\mathbf{p}} \phi_i(\mathbf{p})\mathbf{X}_i, \qquad \mathbf{Y}(\mathbf{p}) = \mathbf{Y}_0 \\
&\text{or} \\
&\mathbf{X}(\mathbf{p}) = \mathbf{X}_0, \qquad \mathbf{Y}(\mathbf{p}) = \mathbf{Y}_0 + \sum_{i=1}^{n_\mathbf{p}} \phi_i(\mathbf{p})\mathbf{Y}_i,
\end{aligned} \tag{14}
$$

or in the set of constant matrices

$$
\mathbf{X}(\mathbf{p}) = \mathbf{X}_0, \qquad \mathbf{Y}(\mathbf{p}) = \mathbf{Y}_0. \tag{15}
$$

In general, since (15) is a subset of (14), a less conservative design is expected when parameter–dependent functions are used. However, in many cases, the improvement in controller performance does not outweigh the larger complexity of the controller implementation algorithm. Therefore, constant Lyapunov functions are usually employed. In this case, and with $\mathbf{B}_2$ and $\mathbf{C}_2$ being constant matrices (see (10)), the computation of the controller matrices in (11) is reduced to a simple linear combination of constant matrices. That is,

$$
\mathbf{A}_c(\mathbf{p}) = \mathbf{A}_{c0} + \sum_{i=1}^{n_p} \phi_i(\mathbf{p})\mathbf{A}_{ci}, \qquad \mathbf{B}_c(\mathbf{p}) = \mathbf{B}_{c0} + \sum_{i=1}^{n_p} \phi_i(\mathbf{p})\mathbf{B}_{ci},
$$
$$
\mathbf{C}_c(\mathbf{p}) = \mathbf{C}_{c0} + \sum_{i=1}^{n_p} \phi_i(\mathbf{p})\mathbf{C}_{ci}, \qquad \mathbf{D}_c(\mathbf{p}) = \mathbf{D}_{c0} + \sum_{i=1}^{n_p} \phi_i(\mathbf{p})\mathbf{D}_{ci},
$$

where $\mathbf{A}_{ci}, \ldots, \mathbf{D}_{ci}$ are constant matrices computed off–line:

$$\begin{aligned}
\mathbf{A}_{ci} &= \mathbf{N}^{-1}(\hat{\mathbf{A}}_i - \hat{\mathbf{B}}_i\mathbf{C}_2\mathbf{Y}_0 - \mathbf{X}_0(\mathbf{A}_i - \mathbf{B}_2\hat{\mathbf{D}}_i\mathbf{C}_2)\mathbf{Y}_0 - \mathbf{X}_0\mathbf{B}_2\hat{\mathbf{C}}_i)\mathbf{M}^{-T}, \\
\mathbf{B}_{ci} &= \mathbf{N}^{-1}(\hat{\mathbf{B}}_i - \mathbf{X}_0\mathbf{B}_2\hat{\mathbf{C}}_i), \\
\mathbf{C}_{ci} &= (\hat{\mathbf{C}}_i - \hat{\mathbf{D}}_i\mathbf{C}_2\mathbf{Y}_0)\mathbf{M}^{-T}, \\
\mathbf{D}_{ci} &= \hat{\mathbf{D}}_i,
\end{aligned}$$

with $\mathbf{M}$ and $\mathbf{N}$ being derived from the factorisation problem[3]

$$\mathbf{I} - \mathbf{X}_0\mathbf{Y}_0 = \mathbf{N}\mathbf{M}^T.$$

Finally, at each sample time t_k, the control action is obtained by means of the following steps:

1. The parameter–dependent functions $\mathbf{A}_c(\mathbf{p})$, $\mathbf{B}_c(\mathbf{p})$, $\mathbf{C}_c(\mathbf{p})$ and $\mathbf{D}_c(\mathbf{p})$ are evaluated at the measured value $\mathbf{p}_k = \mathbf{p}(t_k)$.

2. Then, the control signal is calculated by integration of (11).

3.4 Robustness features

It is worthy to note that the proposed controller exhibits interesting robustness properties. They are due to the constraint on the induced norm of the operator $v \to \tilde{\beta}_d$, as can be shown from (2) – (4) and some block algebra manipulation. In fact, replacing T_r in (2) with (4), it becomes clear that the operator $v \to \hat{P}_g$ differs from $\hat{\beta}_d \to \hat{P}_g$ only in the parameter dependent gains k_v and k_β, respectively. Therefore, if a signal is passed through the filter

$$\frac{k_v(\mathbf{p})}{k_\beta(\mathbf{p})}(s\tau + 1)$$

and entered in the input $\hat{\beta}_d$ of $G(\mathbf{p})$, we obtain the same response that if the signal is entered in the input v of $G(\mathbf{p})$. This fact implies that imposing a constraint on the induced norm of the operator $v \to \tilde{\beta}_d$ is equivalent to imposing a constraint on the induced norm of $u_\Delta \to y_\Delta$ in Fig. 8 , where

$$W_\Delta(s) = \frac{k_v(\mathbf{p})}{k_\beta(\mathbf{p})}(s\tau + 1)W_u(s).$$

[3] In the case that parameter–dependent Lyapunov functions are adopted, the previous matrices as well as the factorisation problem must be computed on–line. The guidelines for the implementation can be found in [18].

By the Small Gain Theorem [28], forcing an induced norm on $u_\Delta \to y_\Delta$ lower than 1, the controller guarantees stability even in the presence of low frequency modelling errors lower than $k_v k_u/k_\beta$ and 100% of error at frequencies higher than ω_u.

Observation The design of conventional linear controllers (typically, PI controllers) capable of stabilising the system and showing satisfactory performance all over the expanded operating region of Fig. 3 is not easy. In fact, because of the wide variety of dynamics (see the range of values for parameters k_β, k_v and B_r in Fig. 4), a single linear controller valid for the whole operating region may be too conservative. To partially cope with this problem, the proportional gain of the PI controller can be varied with the inverse of the input gain k_β. By this proportional gain scheduling, a less conservative design is achieved. However, if certain robustness properties are desired over the whole operating region, the design of the gain scheduled PI controller becomes not trivial again. On the contrary, the proposed LPV design method provides a powerful, simple and familiar tool (that resembles $\mathcal{H}_\infty$ and uses LMI optimisation) to design controllers exhibiting optimal performance over the whole operating region. Moreover, the controllers guarantee a given performance index despite the presence of bounded model uncertainties.

4 Simulation results

In this section we present some results that corroborate the effectiveness of the proposed controller. These simulations were developed using an extended model of the wind turbine that includes also the tower bending dynamics. The realistic wind profile employed in the simulations was generated considering the cyclic fluctuations caused by the spatial distribution of the wind speed field. The mechanical and wind speed models are described in detail in [18].

We considered a three–bladed 2MW wind turbine with total inertia $J_r + J_g = 5.58\mathrm{kgm}^2/\mathrm{s}^2$. The parameters of the weighting functions were tuned as follows: $k_e = 10$, $k_u = 0.25$ and $\omega_u = 10$. We evaluated the use of parameter–dependent and constant Lyapunov functions. In both cases we obtained similar performance levels γ. So, we adopted constant Lyapunov matrices and computed the controller following the procedure described in the previous section.

Power balance and gradient control Fig. 9 shows the response of the wind turbine to a wind speed profile above rated wind speed (Fig. 9a). Initially, the turbine is required to operate at its rated power. Between $t = 20$s and $t = 120$s, the power set–point is reduced to half the rated power, whereas the rate of change of the production power is limited to $\pm$2MW/minute. Fig. 9b shows the evolution of the output power. It is observed that the output power effectively tracks the reference (dotted line). Fig. 9c shows the pitch angle response. It is corroborated that the pitch actuator does not suffer from excessive activity despite the strong turbulence.

Delta control Fig. 10 shows the performance of the wind turbine controller in the delta control mode of operation. In this case, the turbine is required to keep 1MW of power reserve.

Fig. 10a displays the wind speed profile. Fig. 10b shows the evolution of the available power, i.e. of the maximum power that can be captured by the turbine (dot–dashed line), the reference power computed by the centralised wind farm controller (dashed) and the output power P_g (solid line). It is observed that the output power closely follows its reference, so that the turbine may eventually increase its production by 1MW at any time. Finally, Fig. 10d shows the pitch angle response.

5 Conclusions

In this paper, we focused on the control of fixed–speed active stall wind turbines. The pitch control system must be able to accomplish diverse tasks such as power production maximisation, power balance, delta and gradient control. The control design task has been addressed using concepts of gain scheduling and linear parameter–varying systems. In this context, the controller design is formulated as an optimisation problem with LMIs, which is solved using broadly available numerical algorithms. The proposed LPV design procedure provides a powerful and simple tool to design controllers exhibiting robust performance over the whole operating region. Simulations were carried out to assess the controller performance under a realistic wind speed profile. The simulation results show the ability of the controller to achieve power balance, gradient and delta control strategies without excessive pitch activity.

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Tables

Table 1: Variables and parameters of the model referred to the low–speed side of the system

Symbol	Description
Ω_r	Speed of the wind rotor
Ω_g	Speed of the generator
θ_s	Torsion angle of the transmission
T_r	Aerodynamic torque developed on the wind rotor
T_g	Reaction torque of the generator
J_r	Inertia of the wind rotor
J_g	Inertia of the generator
K_s	Stiffness of the transmission
B_s	Damping of the transmission

Figure Captions

Figure 1. Active stall wind turbine configuration

Figure 2. Conventional control strategy

Figure 3. Full range control strategy plotted onto the C_P–curve

Figure 4. Intrinsic damping B_r, and sensitivities k_v and k_β for different power set–points: $100\%P_N$ (solid), $50\%P_N$ (dotted), $0\%P_N$ (dashed)

Figure 5. Model of the pitch angle actuator

Figure 6. Region $\mathcal{P}$ of all possible parameter values

Figure 7. (a) Power feedback control scheme. (b) Augmented plant for the controller synthesis

Figure 8. Model uncertainty representation

Figure 9. Operation with power balance and gradient control

Figure 10. Operation with delta control