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On the stability of dc-dc converters in photovoltaic systems undergoing sliding motions

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Abstract

This paper deals with the dynamic behaviour of different dc-dc switching converters in photovoltaic applications. In particular, the ability of the dc-dc converters to accomplish constant regulation of certain circuit variables is examined. This task is addressed from a variable structure system approach. With this objective, the feasibility of establishing stable sliding regimes on constant state co-ordinate surfaces is investigated. Differences in the dynamic behaviour of the dc-dc converters between photovoltaic and conventional power supply applications are remarked.

***Index Terms*:** Photovoltaic power systems, DC-DC power conversion, switched mode power supplies, Variable structure systems.

1 Introduction

The theory of variable structure systems (VSS) and the associated sliding motions (Utkin 1977, Sira-Ramirez 1988, Hung et al. 1993) provides powerful tools to understand in a simple manner the behaviour of conventional switched-controlled systems, to readily recognise their limitations and to contribute more efficient control solutions. For this reason, several articles analysing the performance of sliding mode controlled electronic circuits are found in literature (Sira-Ramirez 1987, Malesani et al. 1995, Nicolas et al. 1996, Martnez-Salamero et al. 1998).

Dc-dc switching converters are frequently part of photovoltaic conversion systems. Efficiency of solar cells strongly depends on operating conditions (for instance: cells voltage, solar radiation and temperature) (Duffie and Beckman 1991). The electronic converters are therefore used to adjust the operating point of the solar cells according to the control objective. Usually, the goal is tracking the point of optimum power generation for efficiency maximisation. In other cases, regulation below the available power is aimed to avoid battery overcharge.

In this context, some conventional dc-dc conversion topologies for photovoltaic applications are analysed from a VSS approach. The aim of this analysis is to evaluate the ability of the converters to show constant regulation properties of certain circuit variables. To accomplish this task, the feasibility of creating sliding motions on constant state co-ordinate surfaces and their sliding dynamics are investigated. On the other hand, the design of outer loops and their associated slow dynamics (i.e. how the signal references are adjusted to satisfy the power tracking objectives) are beyond the scope of the paper.

In the following section, photovoltaic systems comprising buck, buck-boost and boost converters are introduced. Different sliding motions on constant state variable surfaces are created for each converter topology. Then, stability of ideal sliding mode dynamics, and hence practicability of circuit variables regulation, is investigated. Differences in the dynamic behaviour of the converters between photovoltaic and conventional power supply applications are remarked.

2 System description and main results

2.1 Photovoltaic conversion description

A schematic diagram of the photovoltaic system under consideration is depicted in Figure 1. It comprises a solar array, a dc-dc switching converter, batteries and, eventually, a load.

Solar cells have a highly non-linear voltage - current (V-I) characteristic described by

$$i = I_{ph}(\lambda, T) - I_0 \cdot \left(e^{\frac{v}{V_T(T)}} - 1 \right), \tag{1}$$

where i is the cell output current, v is the cell voltage. I_0 is the reverse saturation current of the pn-junction, T_{ph} is the light-generated cell current proportional to the solar radiation λ. Besides, $V_T = q/AkT$, where q is the electron charge, A is the ideality factor, k is the Boltzmann's constant and T is the temperature. The solar cells are aggregated into a solar array by series/parallel connections. Figure 2 shows the V-I characteristic of the solar array and its dependence on λ and T. In Figure 3, the corresponding voltage - power (V-P) characteristic is depicted for different values of λ and T. These curves present a maximum at an optimum voltage V_m that is extremely sensible to changes in solar radiation and temperature (Duffie and Beckman 1991).

The converter suits the voltage of the solar array at the desired operating point to the voltage of the batteries. In the following, three conventional converter topologies are considered: the buck, buck-boost and boost converters. Figure 4 sketches the equivalent electronic circuits of photovoltaic systems comprising these converters.

2.2 Photovoltaic system with buck converter

Let consider now the electronic circuit shown in Figure 4a. The circuit dynamics is described by:

$$\frac{dx}{dt} = f(x) + g(x) \cdot u, \tag{2}$$

where $x = \begin{bmatrix} i_L & v_C \end{bmatrix}^T$ is the system state, $u \in \{0; 1\}$ denotes the switch position and

$$\begin{aligned} f(x) &= \begin{bmatrix} \frac{v_C - E}{L} & \frac{-i_L + i_S(v_C)}{C} \end{bmatrix}^T, \\ g(x) &= \begin{bmatrix} \frac{v_C}{L} & \frac{i_L)}{C} \end{bmatrix}^T. \end{aligned} \tag{3}$$

Observation: The electronic circuit represented in Figure 4a is a buck converter, being the solar array its power source and the battery its load. Notice, however, that the non-linear state space model (2)-(3) is similar to the classical description of the boost power supply dynamics (in continuous mode), where $-i_S(v_C)$ is the load characteristic (Sira-Ramirez 1987). Hence, the circuit will be viewed, for the dynamic analysis, as a boost converter with constant input voltage E, output voltage v_C and inverted power flow.

2.2.1 Voltage regulation

The case of voltage regulation is firstly analysed. Linear as well as non-linear surfaces involving complete or partial state feedback can be proposed. Among them, the surface defined by

$$\sigma_v = h_v(x) = v_C - V \tag{4}$$

presents interesting features. In fact, (4) has the advantage that the desired output voltage is achieved in finite time (when the sliding regime is established) even before steady state is reached. Moreover, zero regulation error is maintained despite all kind of perturbations provided the sliding mode existence condition holds.

The transversality condition

$$L_g h_v = \frac{i_L}{C} \neq 0 \tag{5}$$

holds for $i_L \neq 0$. Then, the equivalent continuous control is well defined and can be derived from invariance conditions $\begin{cases} \sigma_v = 0 \\ \dot{\sigma}_v = 0 \end{cases}$ (Utkin 1977):

$$u_{eq} = -\frac{L_f h_v}{L_g h_v}\bigg|_{\sigma=0} = 1 - \frac{i_S(v_C)}{i_L}\bigg|_{\sigma=0} = 1 - \frac{i_S(V)}{i_L} \tag{6}$$

The sliding mode existence condition

$$0 < u_{eq} = 1 - \frac{i_S(V)}{i_L} < 1 \tag{7}$$

demands that the solar array current be lower than the inductor one.

The ideal sliding dynamics is obtained by replacing (6) in (2):

$$\frac{di_L}{dt} = \frac{1}{L \cdot i_L} \cdot (i_S(V) \cdot V - E \cdot i_L) \,. \tag{8}$$

It is immediate to show that the unique equilibrium point of the system, placed at

$$p_e = \left(\frac{V}{E} i_C(V), V\right), \tag{9}$$

is stable. Therefore, from (7) and (9), constant voltage regulation by sliding mode is practicable provided the voltage reference satisfies $V > E$.

Figure 5 depicts feasible sliding motions on (4). The V-P and V-I (in dotted line) characteristics are also depicted. Sliding mode exists in the region above the V-I curve, where condition (7) holds. Note that once a sliding regime is established, the solar array voltage is constrained to its reference value whereas the state trajectory evolves toward the equilibrium point.

Usually, the control task is maximising the power conversion efficiency. With this purpose, the solar array voltage has to be maintained at its optimum value V_m. One possible solution to this problem is creating sliding motions on (4) as those shown in Figure 5. The optimum voltage reference V_m is sensible to the slow variations in solar radiation and temperature. Then, an outer loop to adjust the voltage reference according to insolation and temperature fluctuations should be implemented. However, as the paper is only intended to study the fast dynamics of the converter, how the reference voltage is calculated is not reported here.

Remark: Control of the dc-dc converter as power supply, with a resistive load R placed instead of the solar array in Figure 4a, has been studied from a sliding mode approach by (Sira-Ramirez 1987). That paper shows that a sliding regime on (4) for output voltage regulation cannot be maintained due to an unstable inductor current dynamics. Conversely, in the current application, the ideal sliding dynamics is stable thanks of the opposite power flow direction.

2.2.2 Current regulation

Current regulation is analysed here. Then, the following sliding surface can be proposed

$$\sigma_i = h_i(x) = i_L - I = 0 \tag{10}$$

Again, this is not the only conceivable sliding surface. However, it shares the same regulation properties than (4) regarding finite reaching time and robustness.

The equivalent control can be derived from $\begin{cases} \sigma_i = 0 \\ \dot{\sigma}_i = 0 \end{cases}$:

$$u_{eq} = 1 - \frac{E}{v_C}, \tag{11}$$

being well defined for $v_C \neq 0$. Then, the necessary and sufficient condition for sliding mode existence is

$$E < v_C \tag{12}$$

The ideal dynamics is governed by:

$$\frac{dv_C}{dt} = \frac{1}{C \cdot v_C} \cdot \left(i_S(v_C) \cdot v_C - E \cdot I \right). \tag{13}$$

Note that, for this configuration, the supplied power is proportional to the inductor current i_L. Hence, (10) is a suitable surface for power regulation choosing $I = P/E$, being P the power demand. Due to the non-linear V-I characteristic of solar cells, two equilibrium points exist provided the power demand P is lower than the maximum power. Figure 6 shows the V-P characteristic of the solar cell, the sliding surface (10) and the equilibrium points. The equilibrium point (p_l) located at the left of the peak power point is unstable (see in Figure 2 that i_S is almost constant for $v_C < V_m$). On the other hand, the equilibrium point located at the right (p_r) is stable due to a higher negative feedback of the output voltage through the current i_S. Hence, a stable sliding motion can be created on (10) for power regulation. Certainly, if the sliding surface is reached at the right of p_l, the state trajectory will evolve toward the stable equilibrium point p_r. However, reaching the sliding surface at the left of p_r leads to an unstable voltage dynamics, and the sliding motion will be left at $v_C = E$ where (12) does not hold. These possible sliding motions and the region where sliding mode exists are also depicted in Figure 6. Note that if $v_C < E$ at p_l, only stable sliding motions occur.

Remark: Inductor current regulation via sliding mode of the converter in power supply applications (with resistive load instead of the solar cell) is treated in (Sira-Ramirez 1987). That paper shows that stable sliding motions on (10) toward a unique equilibrium point take place on the portion of the sliding surface where existence condition holds. In photovoltaic applications, due to the non-linear V-I characteristic of the solar cells, two equilibrium points exist, and stability of the sliding dynamics depends on the point where the surface is reached.

Observation: Although they are beyond the scope of this paper, different ways to avoid unstable sliding dynamics can be proposed. In fact, sliding motions on auxiliary surfaces can be established until the state trajectory approximates the stable region of surface (10) (Mantz et al. 2001). Another method is to indirectly regulate the inductor current by creating a sliding motion on an alternative surface (for instance, for flat output regulation (Fliess et al. 1995)). In this case, constant current (power) regulation is not achieved when the sliding regime is established but only when steady state is reached. At the cost of this asymptotic approach to the desired current (power) and some loss of robustness, a stable sliding dynamics can be attained independently of the non-minimum phase system behaviour.

2.3 Photovoltaic system with buck-boost converter

Consider the photovoltaic system comprising a buck-boost converter shown in Figure 4b. The circuit dynamics is described by (2), where

$$\begin{aligned} f(x) &= \begin{bmatrix} \frac{v_C}{L} & \frac{-i_L+i_S(v_C)}{C} \end{bmatrix}^T, \\ g(x) &= \begin{bmatrix} \frac{-(v_C+E)}{L} & \frac{i_L}{C} \end{bmatrix}^T. \end{aligned} \tag{14}$$

Observation: Due to the symmetry of the buck-boost cell, the dynamical model of the converter for photovoltaic applications and for power supply applications with a load characteristic $-i_S(v_C)$ (i.e. with inverted power flow) (Sira-Ramirez 1987) are coincident.

2.3.1 Voltage regulation

Again, the creation of sliding motions on the surface

$$\sigma_v = h_v(x) = v_C - V = 0 \tag{15}$$

is analysed first. The equivalent continuous control is in this case given by

$$u_{eq} = -\frac{L_f h_v}{L_g h_v}\bigg|_{\sigma=0} = 1 - \frac{i_S(v_C)}{i_L}\bigg|_{\sigma=0} = 1 - \frac{i_S(V)}{i_L}, \tag{16}$$

and is well defined for $i_L \neq 0$. The sliding mode existence condition

$$0 < u_{eq} = 1 - \frac{i_S(V)}{i_L} \tag{17}$$

demands that the solar array current be lower than the inductor one. The ideal sliding dynamics is obtained by replacing (16) in (2):

$$\frac{di_L}{dt} = \frac{1}{L \cdot i_L} \cdot \left(i_S(V) \cdot V - E \cdot (i_L - i_S(V))\right). \tag{18}$$

It is immediate to show that the unique equilibrium point of the system, placed at

$$p_e = \left(\frac{V+E}{E} i_S(V), V\right) \tag{19}$$

is stable. Therefore, from (17) and (19), voltage regulation by sliding mode is practicable for positive voltage references.

Remark: As it was also shown in (Sira-Ramirez 1987), a sliding regime on (15) for output voltage regulation of a buck-boost power supply cannot be maintained due to an unstable inductor current dynamics. However, due to the opposite power flow direction, the ideal sliding dynamics of the converter in photovoltaic applications is stable.

2.3.2 Current regulation

The sliding surface

$$\sigma_i = h_i(x) = i_L - I = 0 \tag{20}$$

is considered again for current regulation. In this case, the equivalent control is given by

$$u_{eq} = \frac{v_C}{v_C + E}. \tag{21}$$

Then, $v_C > 0$ is a necessary and sufficient condition for sliding mode existence. The ideal dynamics is thus governed by:

$$\frac{dv_C}{dt} = \frac{1}{C \cdot v_C} \cdot \left(i_S(v_C) \cdot v_C - \frac{v_C}{v_C + E} \cdot I \cdot E\right). \tag{22}$$

Note that for this configuration, there is not a direct correspondence between inductor current and supplied power. In fact, the battery and inductor currents are related by the duty cycle of the switch, i.e. the equivalent control (21) (for the correspondence among duty cycle in PWM techniques and equivalent control in sliding mode techniques for the control of electronic switches, the readers are referred to (Sira-Ramirez 1989)).

Inductor current regulation via sliding mode of a buck-boost converter with resistive load (instead of the solar cell) is also treated in (Sira-Ramirez 1987). It is shown there that stable sliding motions on (20) toward a unique equilibrium point take place. In photovoltaic applications, due to the non-linear V-I characteristic of the solar cells, two equilibrium points exist provided the power demand is lower than the available one. Thus, stability of the sliding dynamics depends on the point where the surface is reached. The V-P characteristic of the solar cell, the sliding surface (20) in new co-ordinates, the equilibrium points and possible sliding motions are shown in Figure 7.

2.4 Photovoltaic system with boost converter

Figure 4c shows a boost converter in photovoltaic applications. The circuit dynamics is described by (2, where

$$\begin{aligned} f(x) &= \begin{bmatrix} \frac{v_C}{L} & \frac{-i_L+I_S(v_C)}{C} \end{bmatrix}^T, \\ g(x) &= \begin{bmatrix} \frac{-E}{L} & 0 \end{bmatrix}^T. \end{aligned} \tag{23}$$

Observation: The dynamic model (2)-(23)of this photovoltaic system with boost converter of Figure 4c is similar to the classical description of the buck power supply dynamics, where $-i_S(v_C)$ is the load characteristic (Sira-Ramirez 1987). Hence, the circuit will be viewed, for the dynamic analysis, as a buck converter with constant input voltage E, output voltage v_C and inverted power flow.

2.4.1 Voltage regulation

Voltage regulation by establishing a sliding regime on the surface

$$\sigma_v = h_v(x) = v_C - V = 0 \tag{24}$$

is not feasible for this configuration. In fact, the dynamical system (2) - (23) has relative degree equal to 2. Thus, the transversality condition does not hold, i.e. $L_g h_v = 0$.

Remark: This result is obvious. In fact, the same conclusion has been also obtained for the buck converter as power supply (see (Sira-Ramirez 1987)). Naturally, the inversion of the power flow in photovoltaic applications cannot alter the relative degree of the dynamical system.

2.4.2 Current regulation

For the constant current sliding surface

$$\sigma_i = h_i(x) = i_L - I = 0, \tag{25}$$

the equivalent control is

$$u_{eq} = \frac{v_C}{E}, \tag{26}$$

Then, $v_C < E$ is a necessary and sufficient condition for sliding mode existence. The ideal dynamics is governed in this case by:

$$\frac{dv_C}{dt} = \frac{1}{C} \cdot \left(i_S(v_C) - I\right). \tag{27}$$

There is a unique equilibrium point provided the reference current is lower than the available solar cell current. This equilibrium point is stable due to the negative feedback of the output voltage through the current i_S. Hence, a stable sliding motion can be created on (25) for inductor current regulation whenever the sliding mode existence condition holds.

Remark: Inductor current regulation via sliding mode of this converter working as power supply (i.e. with a resistive load instead of the solar cell) is also treated in (Sira-Ramirez 1987). It is demonstrated there that stable sliding regimes on (25) take place on the portion of the sliding surface where existence condition holds. In photovoltaic applications, the sliding motions toward the equilibrium point are also stable despite the inverted power flow due to the inherent negative voltage feedback in the solar cell.

3 Conclusions

Dynamics of photovoltaic systems comprising different dc-dc converters has been analysed from a sliding mode approach. The dynamic behaviours of these converters in photovoltaic and conventional power supply applications are compared. It is shown that the feasibility of establishing stable sliding motions for voltage and current regulation completely differs among both applications. For instance, unstable sliding dynamics on constant-voltage surfaces for boost and buck-boost power supplies become stable when the load is replaced by a solar array. Regarding current regulation of these converters, two equilibrium points exist owing to the non-linear characteristic of the solar cells. Hence, both stable and unstable sliding motions occur. On the other hand, the possibility (or not) of imposing sliding regimes on constant-current or constant-voltage surfaces for buck power supplies remains unaltered when the load is substituted by the solar array.

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